capture of the K meson. Theory and experiment both indicate that the emission probability of Auger electrons should be small in C,N,O as compared with the nuclei of Ag and Br. In the present experiment it was found that Auger electrons accompanied the K^- capture event in approximately 5% of light nuclear captures (as indicated by the short-prong method) and 38% of the heavy nuclear capture (as indicated by the absence of short prongs). The observed total Auger electron emission intensity was 26.1%, thus confirming the value of 25.5% observed by Grote *et al.*[38] In the wet stack 895 K_σ events were selected for analysis by the short-prong method. Of these, 448 or 50% were observed to have at least one prong in the range 3 μ to 60 μ. This larger percentage of captures in light nuclei as compared with the 32.5% in normal G5 is of course mainly associated with the increased oxygen content of the water-soaked emulsions. If the fraction 88% is used to correct from the number of indicated short-prong events to the total number of light nuclear captures, we then estimate that the fraction of light nuclear captures to all K^- captures in the water-soaked emulsion was 50/18.8∼57%.

PHYSICAL REVIEW VOLUME 129, NUMBER 1 1 JANUARY 1963

Radiative Corrections to $K \to e\nu$ and $K \to \mu\nu$ Decays*

N. P. CHANG
Columbia University, New York, New York
(Received 21 June 1962)

The radiative corrections to the ratio of the $K \to e\nu$ rates to the $K \to \mu\nu$ rates are calculated for an experiment where photon counters are used to help discriminate against the K_{e3} background. This is in contrast to the previously calculated case where all photon energies are included in the rate. The theoretical advantage of including only the low-energy photons is emphasized in this note.

INTRODUCTION

THE $\mu - e$ universality hypothesis will soon be tested in strangeness-nonconserving weak interactions, by means of a comparison of the rate $K \to e\nu$ decay with that of $K \to \mu\nu$ decay. Such a test involves quite critically a knowledge of the radiative effects involved in the experiment, as was already the case with $\pi \to (e,\mu)+\nu$ experiments. The present note is an attempt to calculate the appropriate radiative effects with respect to a particular kind of experiment.

In the $\pi \to e\nu/\mu\nu$ case, calculations[1,2] on the radiative effects have been carried out for an experiment in which all events with electron energy not less than $E_{\max}-\Delta E$ are accepted, regardless of the energy of the accompanying bremsstrahlung photons. This includes, then, photons of maximum available energy. Such a situation is not desirable in the $K \to e\nu/\mu\nu$ case. Here we have a sizeable three-body leptonic decay background, with the neutral pions giving rise to an abundant supply of high-energy photons. This makes it hard to distinguish between the K_{e2} electrons from some of the K_{e3} electrons of comparable energy.

An alternate experiment that we envisage would be as follows. All electron events are to be accepted, provided (1) the accompanying photon bremsstrahlung be of energy $\leq q_1$, and (2) the electron energy is in the range characterized by

$$E_e \equiv \frac{m_K^2+m_e^2}{2m_K}(1-y) \qquad 0 \leq y \leq \frac{2q_1}{m_K}\frac{1-(m_e/m_K)^2}{1+(m_e/m_K)^2}, \quad (1)$$

which is the full range of K_{e2} electron energies consistent with the photon discrimination energy.

The main difference from the previous procedure lies in the fact that even if q_1, the photon discrimination threshold energy, be not too small, the experiment itself, as well as its theoretical interpretation, will still not be affected appreciably—for the high-energy photons from the neutral pions can readily be discriminated against. And, theoretically, the radiative effect is rather insensitive to q_1, so long as it stays small with respect to the K-meson mass—in the K-meson rest frame. Moreover, from a theoretical point of view, this procedure contains less ambiguities insofar as neglecting certain unknown decay amplitudes in the inner bremsstrahlung matrix element, as will be seen below.

In Sec. I, we discuss in some detail the bremsstrahlung process $K \to e\nu\gamma$, with some attention to the general case, where there can be three amplitudes. The inner bremsstrahlung contribution to the rate of K_{e2} is shown in this section. In Sec. II, the virtual photon effects are analyzed. Only those effects which remain in the ratio of $e\nu/\mu\nu$ rates are considered. It is shown that, in fact, even for virtual photons, only q_μ (photon 4-momentum) $\to 0$ region contributes to the $e\nu/\mu\nu$ ratio. The contribution of the three Feynman diagrams which

* This work was supported in part by the U. S. Atomic Energy Commission.
[1] S. M. Berman, Phys. Rev. Letters **1**, 468 (1958).
[2] T. Kinoshita, Phys. Rev. Letters **2**, 477 (1959).

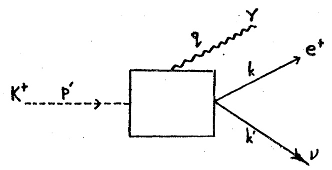

FIG. 1. The diagram contributing to the inner bremsstrahlumg process in which the photon is thrown off from within the box. Note, in particular, the "locality" of the $(e\nu)$ current.

contribute to the $e\nu/\mu\nu$ ratio is written out in Sec. II. In Sec. III, the final result for the $e\nu/\mu\nu$ ratio of rates is written out in full.

I. $K^+ \rightarrow e^+ + \nu + \gamma$

For convenience, we refer explicitly to the electron mode in this and the next section. Similar results for the muon mode can be obtained by substituting m_μ for m_e everywhere.

The most general matrix element for the particular diagram shown is (Fig. 1)

$$(2\pi)^4 i\delta(p'-q-k-k')\epsilon_\mu[f_1\delta_{\mu\lambda}+f_2p_\mu'p_\lambda'+f_3p_\mu'q_\lambda]\times[\bar{u}(k')\gamma_\lambda(1+\gamma_5)v(k)]\frac{1}{i(4p_0'q_0)^{1/2}}, \quad (2)$$

where the three amplitudes f_1, f_2, f_3 are, in general, invariant functions of $-p'\cdot p'$, $p'\cdot q$. In writing down this expression, we omit a term $\epsilon_{\mu\nu\lambda\rho}p_\lambda'q_\rho$, which arises from the vector part of the strangeness-changing current $\langle 0|\mathcal{S}_\lambda^{(V)}|K\rangle$. This is entirely consistent with the principle of minimal electromagnetic coupling, since, in the absence of electromagnetic interactions, this current $\langle 0|\mathcal{S}_\lambda^{(V)}|K\rangle$ does not interact with the lepton current. Even if it were to be included, it would not, on account of the extra q factor, contribute to the final result, as can be seen by the kind of arguments given below.

We may relate these (unknown) amplitudes to the pure decay amplitude by means of well-known arguments of gauge invariance. Thus, if we define the pure decay matrix element by (Fig. 2)

$$(2\pi)^4 i\delta(p'-k-k')\frac{F(-p'\cdot p')}{i(2p_0')^{1/2}}\times[\bar{u}(k')\gamma\cdot p'(1+\gamma_5)v(k)], \quad (3)$$

F, for this decay, is just a constant, since the K meson is on the mass shell. To distinguish it from the general case, we denote $F(m_K^2)$ by f.

Then, we infer, for $q_\mu \rightarrow 0$, that $(e^2/4\pi \sim 1/137)$

$$f_1 \xrightarrow{q\rightarrow 0} ef, \qquad f_2 \xrightarrow{q\rightarrow 0} -2e\left[\frac{\partial F(z)}{\partial z}\right]_{z=m_K^2}. \quad (4)$$

But for the inner bremsstrahlung correction to $K \rightarrow e\nu$ itself, it is precisely the $q_\mu \rightarrow 0$ part of the spectrum that is to be included. Thus, in calculating the real photon process, f_1 may be directly replaced by ef without hesitation. The effect of f_2, f_3 on the real photon process is nil, as can be seen in the K-meson rest frame, where $p'\cdot\epsilon = 0$. However, their effects on the virtual photon process remain to be seen in the next section.

The total inner bremsstrahlung contribution to the $K \rightarrow e\nu$ decay, integrated over all electron energies consistent with the requirement that the bremsstrahlung photon be of energy less than q_1, is given by $(m_K \equiv \mu,\ m_e \equiv m,\ q_1/\mu \ll 1)$

$$\text{Rate (inner brems.)} = \frac{f^2}{4\pi}m^2\mu\left(1-\frac{m^2}{\mu^2}\right)^2\frac{\alpha}{\pi} \times\left\{1+\frac{2m^2}{\mu^2-m^2}\ln\frac{\mu}{m}+\left(2\ln\frac{\lambda}{2q_1}-2\frac{\mu^2+m^2}{\mu^2-m^2}\times\ln\frac{\mu}{m}\ln\frac{\lambda}{2q_1}+1\right)+\frac{\mu^2+m^2}{\mu^2-m^2}\times\left[\ln\frac{\mu}{m}-2L\left(\frac{m^2}{\mu^2}\right)-\frac{\pi^2}{3}-\left(\ln\frac{\mu}{m}\right)^2-4\ln\frac{\mu}{m}\ln\left(1-\frac{m^2}{\mu^2}\right)\right]\right\}, \quad (5)$$

where $L(x)$ is a Spence function[3] defined by

$$L(x) = \int_0^x \frac{\ln(1-t)}{t}dt.$$

λ is the "fictitious" photon mass, which takes care of the infrared divergence in a familiar way.

II. VIRTUAL PHOTON EFFECT

The virtual photon effect needs some discussion. Since we are interested only in the ratio of the $e\nu$ with $\mu\nu$ rates, we need not consider diagrams which will only cancel out in the ratio. Thus, we have to consider the three diagrams as shown by Fig. 3. Of these diagrams, only the second diagram, Fig. 3(b), is affected by f_2, f_3. Now we show that only the $q_\mu \rightarrow 0$ part of f_2, f_3 is important in the $e\nu/\mu\nu$ ratio.

One encounters integrals of the type

$$\int d^4q\frac{1}{q^2-2k\cdot q}\frac{1}{q^2+\lambda^2}f(q^2, p'\cdot q).$$

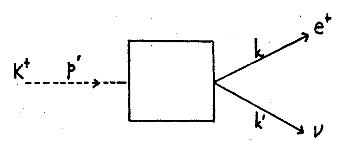

FIG. 2. $K^+ \rightarrow e^+\nu$ pure decay.

[3] K. Mitchell, Phil. Mag. 40, 351 (1949).

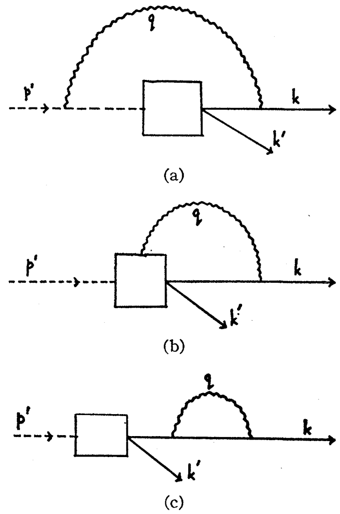

FIG. 3. The three radiative correction perturbation diagrams which contribute to the final $(e\nu)/(\mu\nu)$ ratio.

This being an invariant integral, we can evaluate it, in particular, in the electron rest frame, in which case the integral becomes

$$\int d^4q \frac{1}{\mathbf{q}^2-q_0^2+2mq_0}\frac{1}{q^2+\lambda^2}f(q^2,p'\cdot q).$$

It is immediately evident that one obtains a $\log m$ term from the $q_\mu\to 0$ part of the integration. The infinite q_μ, i.e., the upper limits of integration, yields terms essentially independent of m, since the function $f(q^2,p'\cdot q)$ does not depend on m. Since $q_\mu=0$ is invariant against transformation back to the K-meson rest frame, or, indeed, any other frame, we conclude that the result holds true for all frames.

Hence, we conclude that, even for virtual photon effects, the m-dependent terms come from $q_\mu\sim 0$ region. So long as the f_2, f_3 are smoothly varying functions of q_μ as $q_\mu\to 0$, we can safely replace them by their values at $q_\mu=0$ to obtain the m-dependent terms. In particular, we neglect the f_3 term altogether, since it is multiplied by q_μ. The f_2 term is included in the final result. Meanwhile, we omit explicit mention of f_2.

The contribution of the three virtual diagrams to the rate is evaluated to be

$$\text{Rate (with virtual photons of Fig. 3)}=\frac{f^2}{4\pi}m^2\mu\left(1-\frac{m^2}{\mu^2}\right)^2$$
$$\times\left\{1+\frac{\alpha}{\pi}\left[-\frac{7}{4}-7\ln\frac{\Lambda}{m}-\ln\frac{\lambda}{m}+\frac{11}{2}\ln\frac{\mu}{m}+\frac{8m^2}{\mu^2-m^2}\ln\frac{\mu}{m}+\frac{\mu^2+m^2}{\mu^2-m^2}\left(2\ln\frac{\lambda}{m}\ln\frac{\mu}{m}-\left(\ln\frac{\mu}{m}\right)^2\right)\right]\right\}. \quad (6)$$

In obtaining this result, the structure of the amplitudes $F(-p'\cdot p')$ and $f_2(q^2,p'\cdot q)$ are taken to be constants, while a Feynman cutoff function

$$\Lambda^2/(q^2+\Lambda^2)$$

is introduced where necessary. We discuss this point further in the next section.

III. $(e\nu)/(\mu\nu)$ RATIO

Combining the results of the last two sections, one obtains the total result for the ratio of the $K\to e\nu$ rate with the $K\to\mu\nu$. We write it out, for simplicity, to first order in m_e^2/m_K^2, m_μ^2/m_K^2. Of course, q_1/m_K must still be $\ll 1$.

$$\frac{R(K\to e\nu)}{R(K\to\mu\nu)}=\frac{m_e^2(m_K^2-m_e^2)^2}{m_K^2(m_K^2-m_\mu^2)^2}\left\{1-\frac{2\alpha}{\pi}\left[-\ln\frac{q_1^\mu}{q_1^e}+\left(1+\frac{2m_e^2}{m_K^2}\right)\ln\frac{m_K}{m_e}\ln\frac{m_e}{2q_1^e}-\left(1+\frac{2m_\mu^2}{m_K^2}\right)\ln\frac{m_K}{m_\mu}\ln\frac{m_\mu}{2q_1^\mu}\right.\right.$$
$$+\left(\frac{3}{4}-8\frac{m_e^2}{m_K^2}\right)\ln\frac{m_K}{m_e}-\left(\frac{3}{4}-8\frac{m_\mu^2}{m_K^2}\right)\ln\frac{m_K}{m_\mu}$$
$$+\left(1+\frac{2m_e^2}{m_K^2}\right)\left(\ln\frac{m_K}{m_e}\right)^2-\left(1+\frac{2m_\mu^2}{m_K^2}\right)\left(\ln\frac{m_K}{m_\mu}\right)^2$$
$$\left.\left.+\frac{m_e^2-m_\mu^2}{m_K^2}\left(\frac{\pi^2}{3}-1\right)\right]\right\}. \quad (7)$$

This is the result where the cutoffs have been taken to be the same for both e,μ case. This is certainly justified in the diagrams Fig. 3(a), (b), where one can interpret the cutoff as a structural effect of the K meson. Since it is usually granted that $(e\nu)$ $(\mu\nu)$ occur at a single vertex in weak interactions, the structural effects depend entirely on $(k+k')\cdot(k+k')=p'\cdot p'$, the K-meson mass. That is to say, the cutoffs in such cases do not involve the leptons e,μ. And this is true of, indeed, all the virtual diagrams, except the renormalization diagram, Fig. 3(c). The renormalization diagram involves the intrinsic structure of the electron (muon). Thus, if the intrinsic structure of e,μ be not the same, one should add to the ratio the term

$$R_0\left[-\frac{2\alpha}{\pi}\left(-\frac{1}{4}\ln\frac{\Lambda_\mu}{\Lambda_e}\right)\right]. \quad (8)$$

Such a term is not at all appreciable. For even if Λ_e/Λ_μ be as large as m_μ/m_e, the additional term is numerically $(-0.0062)R_0$, where R_0 is the bare ratio.

Numerically, the radiative effects may be summarized by $(q_1^\mu=q_1^e)$

$$\frac{R(K\to e\nu)}{R(K\to\mu\nu)}=(2.493\times10^{-5})\times\left[1-\frac{2\alpha}{\pi}\left(40.40+5.21\ln\frac{m_e}{2q_1^e}\right)\right]$$
$$\cong(2.493\times10^{-5})\left(1-0.188-0.0242\ln\frac{m_e}{2q_1^e}\right). \quad (9)$$

The major contribution to the 0.188 comes from the giant $[\ln(m_K/m_e)]^2 \sim (6.903)^2$.

We repeat, at this point, that the ratio of rates given here involves similar criterion of acceptance of events in both the e, and the μ experiments.

The effects of f_2 on the total ratio of rates might be mentioned. It replaces the term

$$\frac{3}{4}\left(\ln\frac{m_K}{m_e}-\ln\frac{m_K}{m_\mu}\right)$$

in expression (7) above with

$$\frac{3}{4}\ln\frac{m_\mu}{m_e}\left(1+\frac{4}{3}m_K^2\frac{F'}{f}\right), \tag{10}$$

where

$$\frac{F'}{f}=\frac{dF(z)}{dz}\bigg|_{z=m_K^2}\times\frac{1}{F(m_K^2)}.$$

Because f_2 modifies a term which is not dominant in the total ratio, the effect of f_2 is again not appreciable, even though m_K^2F'/f itself may not be very small compared with unity. A numerical estimate of F' involves strong interactions. We do not do so here, aside from noting that F'/f is typically of order $1/m_N^2$, where m_N denotes the nucleon mass.

For comparison, we quote the corresponding ratio of rates as calculated from Berman's formula (replacing m_π everywhere by m_K) the radiative effect being, for $2\Delta E_\mu/m_e \sim 1$,

$$-0.232+0.027\ln(2\Delta E_e/m_e).$$

This is the case when all real photon momenta are integrated over.

Lastly, it may be of interest to apply the photon discrimination result to the $\pi \rightarrow (e,\mu)+\nu$ case. The numerical result turns out to be

$$-0.154-0.024\ln(m_e/2q_1^e)$$

as compared with Berman's result

$$-0.14+0.021\ln(2\Delta E_e/m_e).$$

Note that, in Berman's case, E_μ is taken to be well resolved, so that $2\Delta E_\mu/m_e \sim 1$, while in the present procedure it is natural that q_1^μ be equal to q_1^e.

CONCLUSION

An experiment which uses photon counters to help discriminate against the background to the $K \rightarrow (e,\mu)+\nu$ is considered in this calculation of the radiative corrections. Theoretically, this procedure is less ambiguous than the previous calculations where all photon momenta are included, the unknown f_2, f_3 amplitudes in the photon bremsstrahlung matrix element being a source of ambiguity in the previous work. Experimentally, perhaps, this photon counter procedure may not be entirely unfeasible, in which case it would be worthwhile carrying out.

ACKNOWLEDGMENT

I wish to acknowledge most gratefully the patient advice and constant encouragement from Professor Gerald Feinberg, who first suggested this problem.